Design of Disturbance Decoupled Bilinear Observers

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An observer structure for bilinear systems is formulated such that the estimation error is independent of unknown external disturbances. The sufficient conditions for the existence of a stable bilinear observer are described. The proposed observer is applied to estimate the tire force in a vehicle semi-active suspension problem.

Key Words: Bilinear Observer, Bilinear Systems, Disturbance, Stability, Semi-Active Suspensions

1. Introduction

A bilinear system is a special nonlinear system. In practice, there exist many bilinear systems such as semi-active suspension systems, ecological systems, nuclear reactor systems, heat exchangers and so on. Existence of bilinear observers has been studied by Grsselli(1981) and Derese(1981). The study on the design of an observer for bilinear systems without unknown disturbances has been made by many authors(Hara, 1976; Williamson, 1977; Funahashi, 1979; Derese, 1979; Hsu, 1981 and Kimbrough, 1984). The design of an observer for a linear system with unknown disturbances has been studied by many people(Bhattacharyya, 1978; Kobayashi et al., 1982; Fairman et al., 1984, etc), and the study of a disturbance decoupled bilinear observer has been made by A. Hac(1989). However, relatively little research has been performed on the state estimation of bilinear systems with an unknown disturbance.

The bilinear observers proposed by former researchers require a necessary condition which makes the observer error dynamics linear, i.e., the error dynamics are independent of the bilinear input. An alternative method proposed by Wiliamson(1977) tends to produce complicated bilinear observer schemes involving strongly nonlinear functions of the input and its higher order derivatives. Derese et al.(1979) proposed another design method by exploiting the boundedness of the input. Their method can be used only when measurements are combinations of the states, i.e., $y=Cx$. A class of bilinear observer presented by Hsu and Karanam(1981) is valid only for a bilinear systems having an exponentially bounded input. The necessary conditions for the disturbance decoupled bilinear observer proposed by Hac(1989) require the measurements of the disturbance related states, that is the states on which the disturbance is applied. However, in general, the measurements of the states coupled with unknown disturbance are very difficult to make.

In this paper an observer for a bilinear system with an unknown disturbance whose estimation error is independent of the disturbance is investigated and the stability of the bilinear observer is discussed. The observer structure proposed in this paper is an extension of observers for linear systems with unknown inputs. The conditions for the bilinear observer discussed in this paper are less restrictive for the measurements because the structure of the error dynamic equation for proposed observer is bilinear, i.e., the proposed bilinear observer is designed such that the norm of the estimation error decays to zero exponentially irrespective of the input while the estimation error dynamics depend on the input. The proposed bilinear observer can be designed for a

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class of time-invariant bilinear systems with bounded inputs. Proofs of theorems are collected in the Appendix.

2. Disturbance Decoupled Billinear Observers

Consider an n-dimensional bilinear system with unknown disturbance $w \in R^q$ represented by

$$\dot{x} = Ax + \sum_{i=1}^{m} D^i x u_i + Fw \tag{1}$$

$$y = Cx + \sum_{i=1}^{m} E^i x u_i + F_y w \tag{2}$$

where x is an n-state vector, $u = [u_1, u_2, \cdots, u_m]^T$ is an m-input vector, y is a p-output vector and all matrices are constant and have proper dimensions. Assume that all inputs are bounded, i.e.,

$$u(t) \in \Omega \subset R^m$$
$$\Omega = \{u(\cdot) \mid \| u(t) \| \leq u_{\max}\}$$

There exists some bound on the input in practical applications.

An intuitive approach to design a state observer is to copy its state Eq. (1) plus a feedback term which utilizes the information contained in the measurement, y. The problems associated with this intuitive approach are due to the fact that the error dynamics of the observer designed by this method depend on the unknown disturbance, w, in addition to the input, u. An oberver structure for bilinear systems with unknown disturbances is formulated to overcome these problems.

In what follows it is assumed that the bilinear system represented by (1)~(2) satisfies the following assumptions.

(I) the number of measurements is greater than the number of unknown disturbances, i.e.,

$$dim(y) = p > dim(w) = q.$$

(II) the number of states on which the disturbance is applied is the same as that of the disturbance.

(III) q outputs are represented as functions of the disturbance related states to which the disturbance is applied.

If the bilinear system represented by Eqs. (1) and (2) can be decomposed into

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \sum_{i=1}^{m} \begin{bmatrix} D_{11}^i & \underline{0} \\ D_{12}^i & D_{22}^i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} u_i + \begin{bmatrix} \underline{0} \\ F_2 \end{bmatrix} w \tag{1'}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_{11} & \underline{0} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \sum_{i=1}^{m} \begin{bmatrix} E_{11}^i & \underline{0} \\ E_{21}^i & \underline{0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} u_i + \begin{bmatrix} F_{y1} \\ \underline{0} \end{bmatrix} w \tag{2'}$$

where $y_2 \in R^q$, $x_2 \in R^q$, $w \in R^q$, $\underline{0}$ is matrix with appropriate dimension and $det(C_{22}) \neq 0$, then the bilinear system satisfies the assumptions.

For the bilinear system (1)~(2) which satisfies the assumptions (I)~(III), a bilinear observer with the following structure is proposed to obtain the state estimate whose estimation error tends to zero asymptotically irrespective of the disturbance, w, and the input, u for all initial conditions, x_0, z_0.

$$\dot{z} = A_z z + \sum_{i=1}^{m} D_z^i z u_i + Ly + H[P_2 y - y_z] \tag{3}$$

$$y_z = C_z z + \sum_{i=1}^{m} E_z^i z u_i \tag{4}$$

$$v = B_z z + \sum_{i=1}^{m} B_u^i z u_i + P_1 y \tag{5}$$

where $v \in R^q$, $z \in R^r$, $(r+q) \leq n$, $y_z \in R^s$, $s \leq p$ and all matrices are constant with proper dimension. It should be noted that the dimension of v is q which is the same as that of the unknown disturbance w. This and Eq. (5) imply that q states, v, is estimated by a combination of measurements, y, input, $u(t)$, and estimated states, z. The bilinear observer represented by (3)~(5) is said to be an observer for $Kx (K \in R^{(q+r) \times n})$ of the bilinear system (1)~(2), if

$$\lim_{t \to \infty} \frac{d^i}{dt^i} e(t) = 0, \ (i = 0, 1, 2, \cdots) \tag{6}$$

$$e(t) = \begin{bmatrix} v(t) \\ z(t) \end{bmatrix} - [Kx(t)] \tag{7}$$

independent of the input u, initial state x_0 and z_0 and unknown disturbance(Hara, 1976). If $K = I_n$, it is said to be a state observer. It should be noted that the proposed bilinear observer requires no knowledge of the unknown disturbance w.

The sufficient conditions for the existence of a stable bilinear observer with the structure represented by Eqs. (3) to (5) is given by Theorem 1 and Theorem 2.

Theorem 1. For every input $u \in \Omega$ and every disturbance w, z converges to Wx if there exist some H such that the following conditions are satisfied :

$$(WA - A_z W - LC) - H(P_2 C - C_z W) = \underline{0} \quad (8)$$

$$(WD^i - D_z^i W - LE^i) - H(P_2 E^i - E_z^i W) = \underline{0}, \quad i = 1, \cdots, m \quad (9)$$

$$WF - LF_y - H(P_2 F_y) = \underline{0} \quad (10)$$

and

the bilinear system $\dot{z} = A_e z$ *is asymptotically stable* for *all* $u \in \Omega$ (11)

where

$$A_e = (A_z - HC_z + \sum_{i=1}^{m} D_z^i u_i - \sum_{i=1}^{m} HE_z^i u_i)$$

Remark 1. The condition (11) implies that (A_z, C_z) should be an observable pair.

Theorem 2. For every $u \in \Omega$ and every disturbance w, v converges to $Ux (U \in R^{q \times n})$ as z converges to Wx if:

$$P_1 F_y = \underline{0} \quad (12)$$

$$P_1 C = U - B_z W \quad (13)$$

$$P_1 E^i = -B_u^i W, \; i = 1, \cdots, m. \quad (14)$$

For the class of bilinear systems satisflying the required assumptions (I)~(III), next prodedure can be used for the synthesis of the disturbance decoupled bilinear observer with the structure represented by Eqs. (3) to (5). The observer matrices are determined from the system Eqs. (1) ~(2) and only the observer feedback matrix H is the design parameter.

The synthesis of the distrubance decoupled bilinear observer can be summarized as follows.

(i) Pick v as an estimate of the disturbance related states. The disturbance related states are state variables whose derivatives are affected by the disturbance. For example, let $v = \hat{x}_2 = \widehat{Ux}$ for the system represented by equation. (1′).

(ii) Obtain relations between the disturbance related states x_w i.e. x_2 in equation (1′), the remaining states and the measurement y such that x_w can be written as follow :

$$x_w = f_1(x, u, y) \quad (15)$$

This is possible if the bilinear system represented by Eqs. (1)~(2) satisfies the requirements (I) ~(III).

(iii) Define $x_z = Wx$, which is a subspace of x, from equation (15) and system Eqs. (1)~(2) such that x can be replaced by x_z in Eq. (15),

$$x_w = f_1(x_z, u, y), \quad (16)$$

then v can be written as follow :

$$v = f_1(z, u, y) \quad (17)$$

v is the estimate of the disturbance related states x_w and z is the estimate of the states x_z, i.e., $v = \hat{x}_w$, $z = \hat{x}_z$.

After (iii) we find B_z, B_u^i and P_1 from the system Eqs. (1)~(2).

(iv) Obtain the state equation for z, i.e., A_z, D_z^i, P_2, C_z, E_z^i and L from the bilinear system equation.

(v) Select the observer feedback matrix H to guarantee the stability of the homogeneous part of the error dynamics, i.e., $\dot{e}_z = A_e(u(\cdot))e_z$.

In the design of the bilinear observer the stability of the observer depends on the selection of the observer feedback matrix H. For the bilinear system which satisfies the assumptions (I)~(III), i.e., which can be decomposed into the Eqs. (1′) to (2′) an observer structure which satisfies the Eqs. (8)~(10) and (12)~(14) can be obtained by applying the above procedure. Hence, H should be designed so that the bilinear dynamic system

$$\dot{z} = A_e(u)z = (A_z + HC_z)z + \sum_{i=1}^{m} (D_z^i - HE_z^i) u_i z \quad (18)$$

is asymptotically stable. The observer feedback gain H should be chosen such that the minimum eigenvalue of $Q(u)$ for all $u \in \Omega$ is greater than zero, i.e.,

$$\min_{u \in \Omega} \lambda_{\min}[Q(u)] > 0 \quad (19)$$

where

$$Q(u) = -(A_e^T(u)P + PA_e(u)) \quad (20)$$

$$A_e(u)=(A_z+HC_z)+\sum_{i=1}^{m}(D_z^i-HE_z^i)u_i \quad (21)$$

and P is a positive definite matrix.

The stability guaranteed region of the gain H may be found either analytically or numiercally. The problem of finding the stability guaranteed region of the oberver gain H can be handled separatedly for each application. There is no simple way to do this systematically for any bilinear system.

As an application, the design of a disturbance decoupled bilinear observer for a semi-active suspension is explained in the next section.

3. The Design of a Bilinear Observer for a Semi-active Suspension

3.1 Semi-active suspensions

Consider the quarter semi-active suspension model shown in Fig. 1. The equation of motion of this system is represented by a bilinear form(Yi and Hedrick, 1993):

$$\dot{x}=Ax+Dxu+Fw \quad (22)$$

where the unknown disturbance $w(=\dot{z}_r)$ is the rate of change of road elevation and

$$x=[z_s-z_u \;\; \dot{z}_s \;\; z_u-z_r \;\; \dot{z}_u]^T \quad (23)$$

$$A=\begin{bmatrix} 0 & 1 & 0 & 1 \\ -\frac{k}{m_s} & -\frac{b}{m_s} & 0 & \frac{b}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_u} & \frac{b}{m_u} & -\frac{k_t}{m_u} & -\frac{b}{m_u} \end{bmatrix},$$

$$D=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{m_s} & 0 & \frac{1}{m_s} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_u} & 0 & -\frac{1}{m_u} \end{bmatrix},$$

$$F=\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}. \quad (24)$$

It is assumed that dynamic tire force is proportional to tire deflection by the constant k_t. Also the assumption that tire damping is insignificant is used in the modeling of the semi-active suspension. The semi-active force is implemented by a controllable shock absorber. For this representation, the control input u is the controllable damping rate and is determined by a semi-active control strategy within a given range. In actuality, the variable damping rate, $u(t)$, is modulated in the following admissible space:

$$u=\{u \mid u_{\min}\leq u\leq u_{\max}\} \quad (25)$$

3.2 A billinear observer for a semi-active suspension

Based on the bilinear observer proposed in section 2 an observer is designed to estimate the dynamic tire force, which is difficult to measure in real time. Assume that axle acceleration and sprung mass acceleration are measured. Thus the measurement y is

$$y=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} \dot{x}_2 \\ \dot{x}_4 \end{bmatrix} \quad (26)$$

and matrices C, E^i and F_y are determined by the state Eq. (22). Select the tire deflection as ν, which is a disturbance related state. i.e.,

$$\nu=\hat{x}_3=U\hat{x}$$
$$U=[0\,0\,1\,0] \quad (27)$$

From the relation between $\nu(=\hat{x}_3)$ and the measurement y_2, z is determined, i.e.,

$$\nu=\hat{x}_3=f_1(y_2,\ \hat{x}_1,\ \widehat{(x_2-x_4)})$$
$$=f_2(y_2,\ z) \quad (28)$$

and

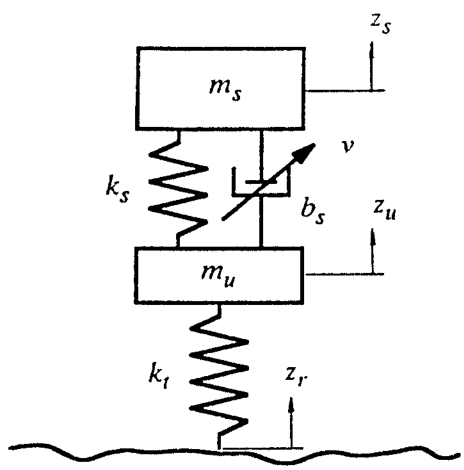


Fig. 1 Quarter car model with semi-active suspension

$$z = W\hat{x} = \begin{bmatrix} \hat{x}_1 \\ (\hat{x}_2 - \hat{x}_4) \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \tag{29}$$

In this case a bilinear observer for a semi-active suspension is expressed as

$$\nu = \hat{x}_3 = \begin{bmatrix} \frac{k}{k_t} & \frac{b}{k_t} \end{bmatrix} z + \begin{bmatrix} 0 & \frac{1}{k_t} \end{bmatrix} zu + \begin{bmatrix} 0 & -\frac{m_u}{k_t} \end{bmatrix} y$$

$$\dot{z} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m_s} & -\frac{b}{m_s} \end{bmatrix} z + \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{m_s} \end{bmatrix} zu + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} [y_1 - y_{1z}] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y_2 \tag{30}$$

where

$$y_{1z} = -\frac{k}{m_s} z_1 - \frac{b}{m_s} z_2 - \frac{1}{m_s} z_2 u \tag{31}$$

Define the estimation error, e_z, as $Wx - z$. Then, the error dynamics are expressed as

$$\dot{e}_z = \begin{bmatrix} h_1 \frac{k}{m_s} & 1 + h_1 \frac{(b+u)}{m_s} \\ (h_2 - 1)\frac{k}{m_s} & (h_2 - 1)\frac{(b+u)}{m_s} \end{bmatrix} e_z = A_e(u) e_z \tag{32}$$

It is straightforward to verify that the bilinear observer to estimate the dynamic tire force for a semi-active suspension satisfies the conditions (8) ~(10) except the condition (11). Therefore if the error dynamics are asymptotically stable, the dynamic tire force estimation error tends to zero by Theorem 1 and Theorem 2. The stability of the bilinear observer depends on the observer feedback gains (h_1, h_2). The stability region of the gains (h_1, h_2) can be found by following the procedure for the gain H given in section 2.

Remark 2. The bilinear system (32) is asymptotically stable if the observer feedback gains (h_1, h_2) satisfy the following conditions :

$$-\frac{4m_s}{u_{max}} \left[1 + 2\frac{(b + u_{min})}{u_{max}} + 2\sqrt{\frac{(b+u_{min})}{u_{max}} \left(1 + \frac{(b+u_{min})}{u_{max}} \right)} \right] < h_1 < 0$$

$$h_2 < 1 \qquad \square$$

The bilinear observer discussed in this section for a semi-active suspension estimates the dynamic tire force, the spring deflection and the spring deflection rate with the axle acceleration and the sprung mass acceleration measurements. The suspension velocity should be known in order to determine the damping rate of the controllable shock absorber. The measurements of acceleration may be made with ease compared to velocity or deflection measurements. As mentioned in the introduction, this study has been motivated by a state estimation problem in semi-active suspension control to reduce the dynamiic axle load where the dynamic tire force and the spring deflection rate are the most important states in the control law(Yi and Hedrick, 1989). Therefore the bilinear observer designed in this section may be very effective in semi-active suspension control to reduce the dynamic axle load.

3.3 Simulation results

The bilinear observer designed in section 3.2 was implemented on a semi-active suspension simulation. To simulate the observer behavior for the semi-active suspension system, the following parameters adapted from heavy truck data(Yi and Hedrick, 1989) were used.

$m_s = 7530[\text{Kg}]$, $m_u = 656.6[\text{Kg}]$
$k = 1786205[\text{N/m}]$, $k_t = 3586419[\text{N/m}]$
$b = 102128[\text{N/(m/s)}]$
$u_{max} = 5 \times b[\text{N/(m/s)}]$
$u_{min} = -0.8 \times b[\text{N/(m/s)}]$

A sum of sine functions was used to generate an unknown road input disturbance.

Figure 2 shows the comparisons between the

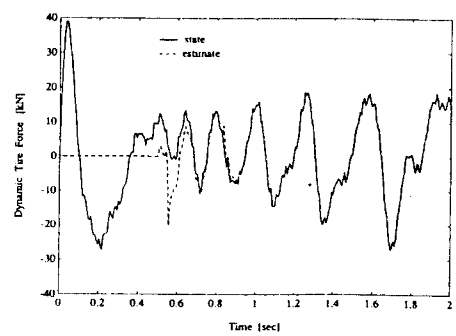
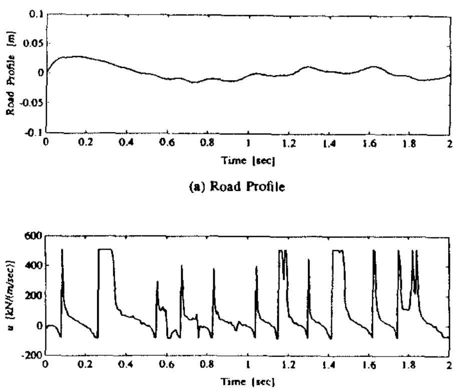


Fig. 2 Comparisons between state and estimated state

actual states and estimated ones. The estimator starts to work at 0.5 second. The estimation error tends to zero as time increases even if the road input disturbance is unknown. The error due to unknown initial conditions converges rapidly and the estimates track the actual state after the transient phase. The unknown road profile and the controlled damping rate are shown in Fig. 3.

The effect of parametric error on the estimation errors was investigated. The actual states and the estimated ones when there is 20% sprung mass error, −10% unsprung mass error, 10% tire stiffness error and −5% suspension stiffness error are compared in Fig. 4. It is shown that the estimation errors are bounded. Fig. 4 indicates that the bilinear observer is robust to parametric error.

Although measurments of the sprung mass velocity may be made by integrating the output of an accelerometer, measurement of the tire forces is very difficult to make for real time control because of the unknown road input. Thus it is beneficial in semi-active suspension control to design an observer which estimates the necessary states, whose estimation error due to initial conditions converges to zero sufficiently quickly and whose error is independent of the unknown road input.



(a) Road Profile

(b) Controlled Damping Rate, $u(t)$

Fig. 3 Unknown road profile(disturbance) and control input u(t)

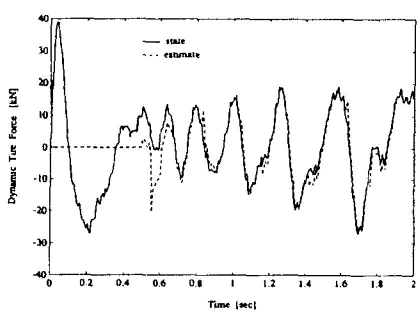


Fig. 4 Comparisons between state and estimated state(with parametric errors : ems = 1.20 emu = 0.9 ekt = 1.1 eks = 0.95)

4. Conclusions

A bilinear observer whose estimation error is independent of the unknown disturbance has been provided and stability conditions for the observer were investigated. The necessary conditions for the measurements are relaxed compared to that of the disturbance decoupled bilinear observer that cancels the effect of the input in the observer internal model.

The proposed disturbance decoupled bilinear observer is applied to a semi-active suspension system. The stability guaranteed region for the observer feedback gain in this case was provided. The dynamic tire force, the spring deflection and the spring deflection rate were estimated without estimation error using the axle acceleration and the sprung mass acceleration measurements which may be made with ease.

In practice, the measurements are contaminated by noise and the optimization of the observer feedback gains in the stochastic case is necessary in addition to the stabilization of the bilinear observers. The optimal bilinear observer design in the stochastic case will be studied in the future.

Appendix

PROOF of THEOREM 1. Define the estimation error, e_z, as :

$$e_z = Wx - z \tag{A.1}$$

then,

$$\dot{e}_z = A_e e_z + \{(WA - A_z W - LC) - H(P_2 C - C_z W)\}x + \sum_{i=1}^{m}\{(WD^i - D_z^i W - LE^i) - H(P_2 E^i$$

$-E_z^i W)\}xu_i$

$$(WF - LF_y - HP_2F_y)w \quad \text{(A.2)}$$

If conditions (8)~(10) are satisfied, then

$$\dot{e}_z = A_e e_z \quad \text{(A.3)}$$

and under the conditions (11) d_z converges to zero asymptotically. □

PROOF of THEOREM 2. By substituting y into Eq. (5) v can be expressed by the following equation :

$$\begin{aligned} v &= B_z z + \sum_{i=1}^{m} B_u z u_i + P_1 y \\ &= Ux + B_z(z - Wx) + \sum_{i=1}^{m} B_u^i (z - Wx) u_i \\ &\quad + P_1 F_y W + (P_1 C - U + B_z W)x \\ &\quad + \sum_{i=1}^{m} (P_1 E^i + B_u^i W) x u_i \end{aligned} \quad \text{(A.4)}$$

Define the error, e_v, as the difference between v and the transformed state Ux :

$$e_v = v - Ux \quad \text{(A.5)}$$

then,

$$\begin{aligned} e_v &= (B_z + \sum_{i=1}^{m} B_u^i u_i) e_z + P_1 F_y w \\ &\quad + (P_1 C - U + B_z W)x \\ &\quad + \sum_{i=1}^{m} (P_1 E^i + B_u^i W) x u_i \end{aligned} \quad \text{(A.6)}$$

If conditions (12) to (14) are satisfied, then

$$e_v = (B_z + \sum_{i=1}^{m} B_u^i u_i) e_z \quad \text{(A.7)}$$

and e_v converges to zero as e_z converges to zero. □

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